Inequalities

So far we learned that equalities are comparisons in which the left side of the equation equals the right side of the equals sign. Inequalities are comparisons where the left side of an equation does not equal to the right side.

For example:

$-3 \neq 6$
$-1 > -9$ → All inequalities
$3 < 7$

Solving inequalities is very similar to solving equalities but your answer will be a solution set often with more than one number as the answer.

Example #1: (left side / right side of equation)

$x + 3 < 2$; $x \in I$ ← symbol means is a member of the set

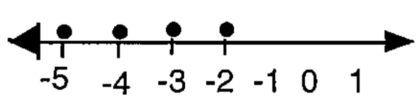
$\underline{-3 \qquad\qquad -3}$ x can only be integers.

$x < -1$ so the solution for x is any integer less than -1.

The answer would be $\{\ldots -4, -3, -2\}$ Infinite set

Sometimes you will need to graph your solutions as number lines.

Graph for the above question:



Thickened arrow shows pattern continues in this direction and dots show only these numbers belong

Infinite set

Example #2: left side right side

$2x + 4 - x \geq 7$; $x \in W$ ← answers must be whole numbers

* simplify each side of equation

$$x + 4 \geq 7 ; x \in W$$

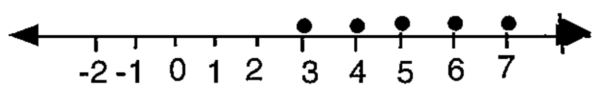
* collect variables then isolate variable

$$\begin{array}{rcl} x + 4 & \geq & 7 \\ -4 & & -4 \\ \hline x & \geq & 3 \end{array}$$

solution is all whole numbers greater than or equal to 3

* { 3, 4, 5 ...}

graph



Infinite Set

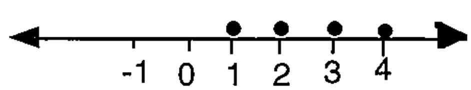
Example #3: $y + 7 + 10 > 15$; $y \in N$

$$\begin{array}{rcl} y + 17 & > & 15 \\ -17 & & -17 \\ \hline y & > & -2 \end{array}$$

y is a member of the set of natural numbers

y is any natural number bigger than -2

{ 1, 2, 3 ... }

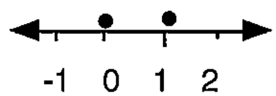


Example #4: $z + 10 < 12;\ z \in W$

$$\begin{array}{lcr} z + 10 & < & 12 \\ -10 & & -10 \\ \hline z & < & 2 \end{array}$$

z is any whole number less than 2

{0, 1 }
Finite set



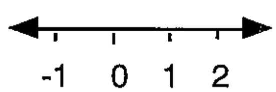
no arrows thickened because finite set

Example #5: $z + 10 < 8;\ z \in W$

$$\begin{array}{lcr} z + 10 & < & 8 \\ -10 & & -10 \\ \hline z & < & -2 \end{array}$$

No whole number is less than -2 so this is a null set

{ $\varnothing$ }
phi



No dots or anything on graph

Solving Inequalities by multiplication or division

-5 < 2

If we multiply each side by 2, this is still true

2 (-5) < 2 (2)

-10 < 4

If we times each side by -2 this happens:

(-2) (-5) (-2)(2)

10 < -4 X NOT TRUE

so you can see that when you multiply each side of an equation by a negative, to make the statement true you must reverse the inequality. (same is true for division)

Therefore, reverse so 10 > -4

More Examples:

#1

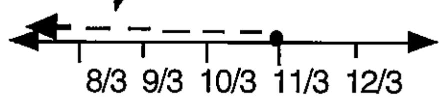
$3p - 7 \le 4$; $p \in Q$ ← means rational numbers

$\underline{\quad +7 \quad +7}$

$\frac{3p}{3} \le \frac{11}{3}$

$\boxed{p \le \frac{11}{3}}$

Broken line indicates only the rational and not the irrational #'s between are included.



The dot over 11/3 means it is part of the solution and the arrow at the end shows the pattern continues forever.

#2

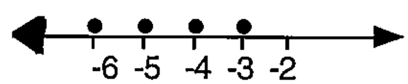
$-6z + 2 > 14$; $z \in I$

$\underline{\quad -2 \quad -2}$

$\frac{-6z}{-6} > \frac{12}{-6}$ dividing by a negative, so to make this a true statement you must reverse the inequality!

$\boxed{z < -2}$

{ ... -5, -4, -3 }



#3

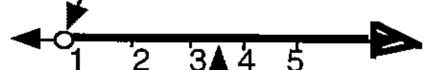
$-9p + 6 < -3$; $p \in R$ ← means real numbers

$\underline{\quad -6 \quad -6}$

$\frac{-9p}{-9} < \frac{-9}{-9}$ ← (reverse the inequality since divided by a negative to solve)

$\boxed{p > 1}$

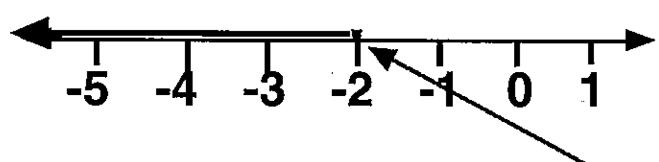
Circle means the closest possible # to one is included in the set but one is not.



The entire line is thickened to show all of the possible #'s from one onward belong to the set of real (R) numbers. The thick arrow on the end means the trend continues forever in that direction.

#4 $-3z-2 \geq z+6 \; ; \; z \in R$

$$
\begin{array}{rcl}
-3z-2 & \geq & z+6 \\
+3z & & +3z \\
\hline
-2 & \geq & 4z+6 \\
-6 & & -6 \\
\hline
-8 & \geq & 4z \\
\div 4 & & \div 4 \\
\hline
-2 & \geq & z
\end{array}
$$



The dot over the -2 indicates that it is included in the solution set and the thickened line and arrow shows that all numbers from -2 to infinity are included in the set also.